

1. An urban highway has a speed limit of 50 km h^{-1} . It is known that the speeds of vehicles travelling on the highway are normally distributed, with a standard deviation of 10 km h^{-1} , and that 30% of the vehicles using the highway exceed the speed limit.

(a) Show that the mean speed of the vehicles is approximately 44.8 km h^{-1} .

(3)

The police conduct a “Safer Driving” campaign intended to encourage slower driving, and want to know whether the campaign has been effective. It is found that a sample of 25 vehicles has a mean speed of 41.3 km h^{-1} .

(b) Given that the null hypothesis is

H_0 : the mean speed has been unaffected by the campaign

State H_1 , the alternative hypothesis.

(1)

(c) State whether a one-tailed or two-tailed test is appropriate for these hypotheses, and explain why.

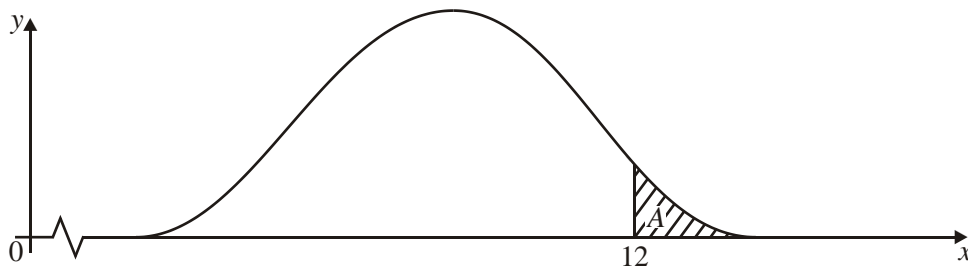
(2)

(d) Has the campaign had significant effect at the 5% level?

(4)

(Total 10 marks)

2. The graph shows a normal curve for the random variable X , with mean μ and standard deviation σ .



It is known that $p(X \geq 12) = 0.1$.

(a) The shaded region A is the region under the curve where $x \geq 12$. Write down the area of the shaded region A .

(1)

It is also known that $p(X \leq 8) = 0.1$.

(b) Find the value of μ , explaining your method in full. (5)

(c) Show that $\sigma = 1.56$ to an accuracy of three significant figures. (5)

(d) Find $p(X \leq 11)$. (5)
(Total 16 marks)

3. The heights of boys at a particular school follow a normal distribution with a standard deviation of 5 cm. The probability of a boy being shorter than 153 cm is 0.705.

(a) Calculate the mean height of the boys.

(b) Find the probability of a boy being taller than 156 cm.

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(Total 6 marks)

4. Bags of cement are labelled 25 kg. The bags are filled by machine and the actual weights are normally distributed with mean 25.7 kg and standard deviation 0.50 kg.

(a) What is the probability a bag selected at random will weigh less than 25.0 kg? (2)

In order to reduce the number of underweight bags (bags weighing less than 25 kg) to 2.5% of the total, the mean is increased without changing the standard deviation.

(b) Show that the increased mean is 26.0 kg. (3)

It is decided to purchase a more accurate machine for filling the bags. The requirements for this machine are that only 2.5% of bags be under 25 kg and that only 2.5% of bags be over 26 kg.

(c) Calculate the mean and standard deviation that satisfy these requirements. (3)

The cost of the new machine is \$5000. Cement sells for \$0.80 per kg.

(d) Compared to the cost of operating with a 26 kg mean, how many bags must be filled in order to recover the cost of the new equipment? (3)

(Total 11 marks)

5. A box contains a large number of biscuits. The weights of biscuits are normally distributed with mean 7 g and standard deviation 0.5 g.

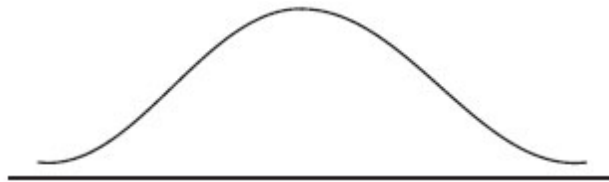
(a) One biscuit is chosen at random from the box. Find the probability that this biscuit

- (i) weighs less than 8 g;
- (ii) weighs between 6 g and 8 g.

(4)

(b) Five percent of the biscuits in the box weigh less than d grams.

- (i) Copy and complete the following normal distribution diagram, to represent this information, by indicating d , and shading the appropriate region.



- (ii) Find the value of d .

(5)

- (c) The weights of biscuits in another box are normally distributed with mean μ and standard deviation 0.5 g. It is known that 20% of the biscuits in this second box weigh less than 5 g.

Find the value of μ .

(4)

(Total 13 marks)

6. In a large school, the heights of all fourteen-year-old students are measured.

The heights of the girls are normally distributed with mean 155 cm and standard deviation 10 cm.

The heights of the boys are normally distributed with mean 160 cm and standard deviation 12 cm.

- (a) Find the probability that a girl is taller than 170 cm.

(3)

- (b) Given that 10% of the girls are shorter than x cm, find x .

(3)

- (c) Given that 90% of the boys have heights between q cm and r cm where q and r are symmetrical about 160 cm, and $q < r$, find the value of q and of r . (4)

In the group of fourteen-year-old students, 60% are girls and 40% are boys.
The probability that a girl is taller than 170 cm was found in part (a).
The probability that a boy is taller than 170 cm is 0.202.

A fourteen-year-old student is selected at random.

- (d) Calculate the probability that the student is taller than 170 cm. (4)

- (e) Given that the student is taller than 170 cm, what is the probability the student is a girl? (3)
(Total 17 marks)

7. In a country called *Tallopia*, the height of adults is normally distributed with a mean of 187.5 cm and a standard deviation of 9.5 cm.

- (a) What percentage of adults in *Tallopia* have a height greater than 197 cm? (3)

- (b) A standard doorway in *Tallopia* is designed so that 99% of adults have a space of at least 17 cm over their heads when going through a doorway. Find the height of a standard doorway in *Tallopia*. Give your answer to the nearest cm. (4)
(Total 7 marks)

8. It is claimed that the masses of a population of lions are normally distributed with a mean mass of 310 kg and a standard deviation of 30 kg.

- (a) Calculate the probability that a lion selected at random will have a mass of 350 kg or more.

(2)

- (b) The probability that the mass of a lion lies between a and b is 0.95, where a and b are symmetric about the mean. Find the value of a and of b .

(3)

(Total 5 marks)

9. The heights, H , of the people in a certain town are normally distributed with mean 170 cm and standard deviation 20 cm.

- (a) A person is selected at random. Find the probability that his height is less than 185 cm.

(3)

- (b) Given that $P(H > d) = 0.6808$, find the value of d .

(3)

(Total 6 marks)

10. The speeds of cars at a certain point on a straight road are normally distributed with mean μ and standard deviation σ . 15% of the cars travelled at speeds greater than 90 km h^{-1} and 12% of them at speeds less than 40 km h^{-1} . Find μ and σ .

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(Total 6 marks)